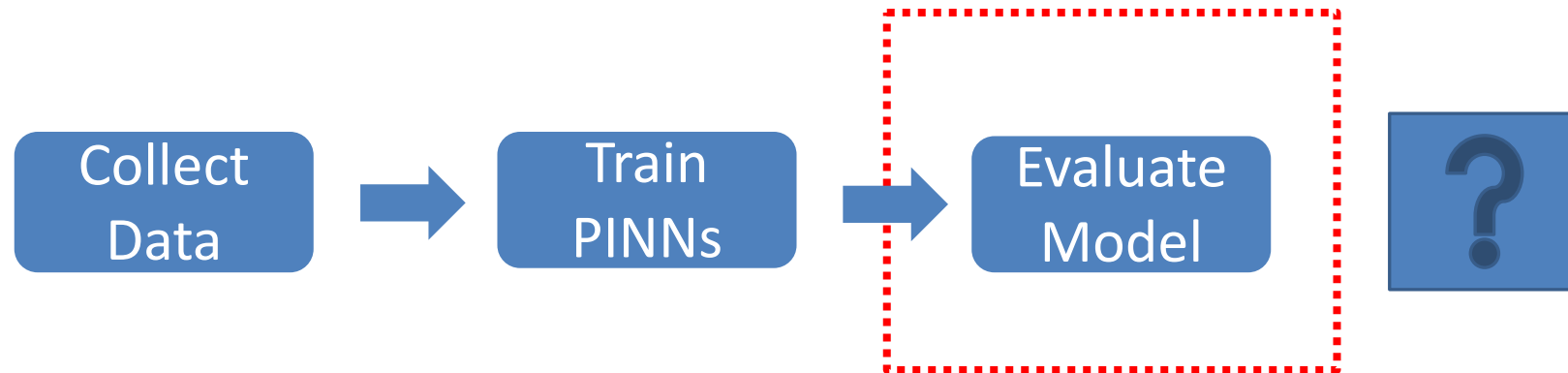# Verifying PINNS

SHAURYA GOMBER
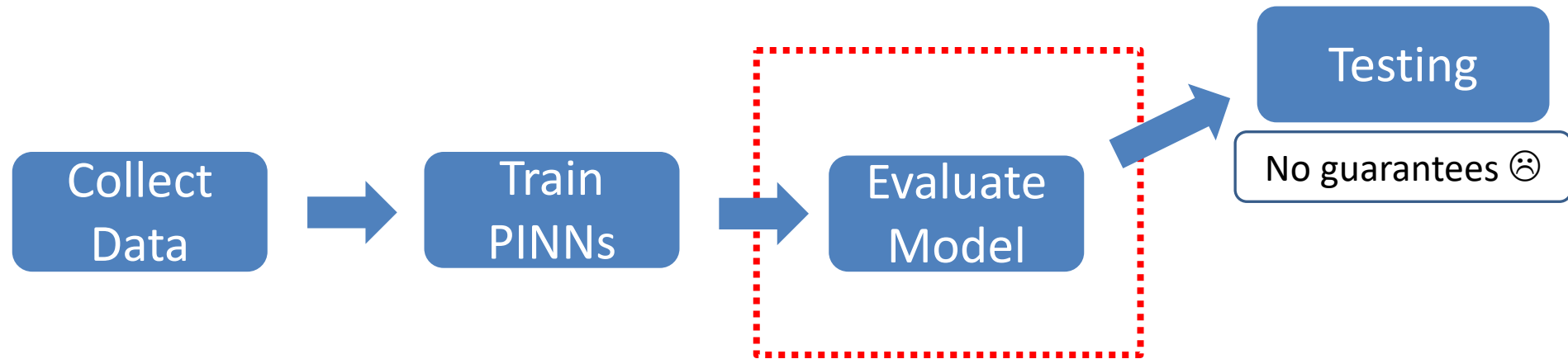
AVALJOT SINGH

DEBANGSHU BANERJEE
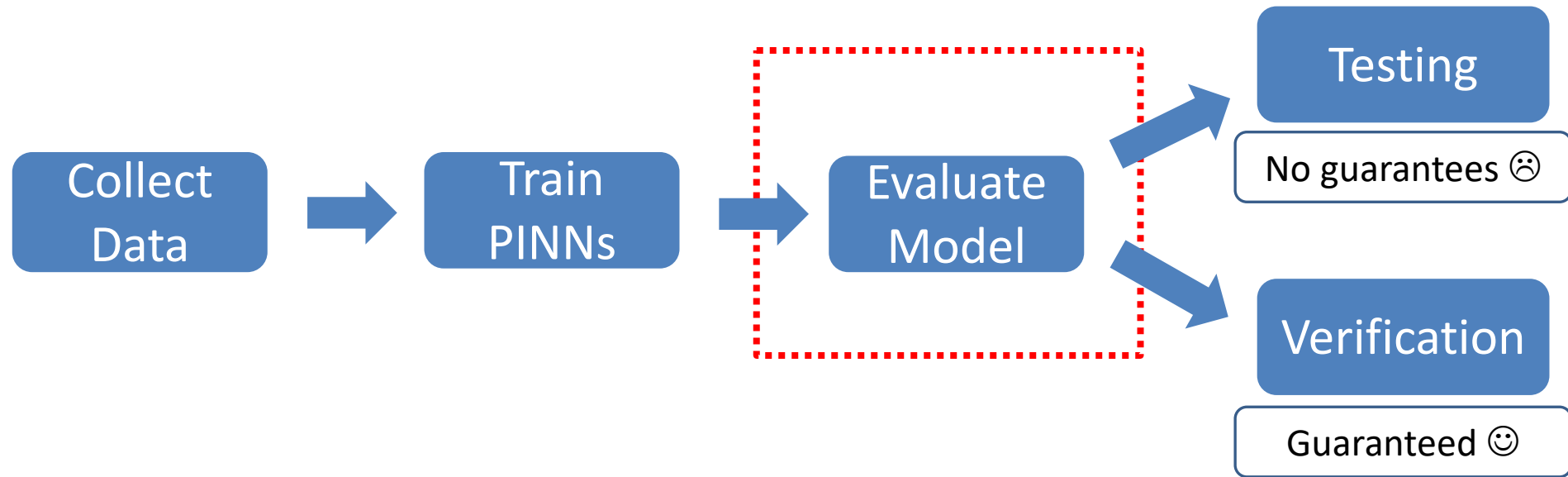
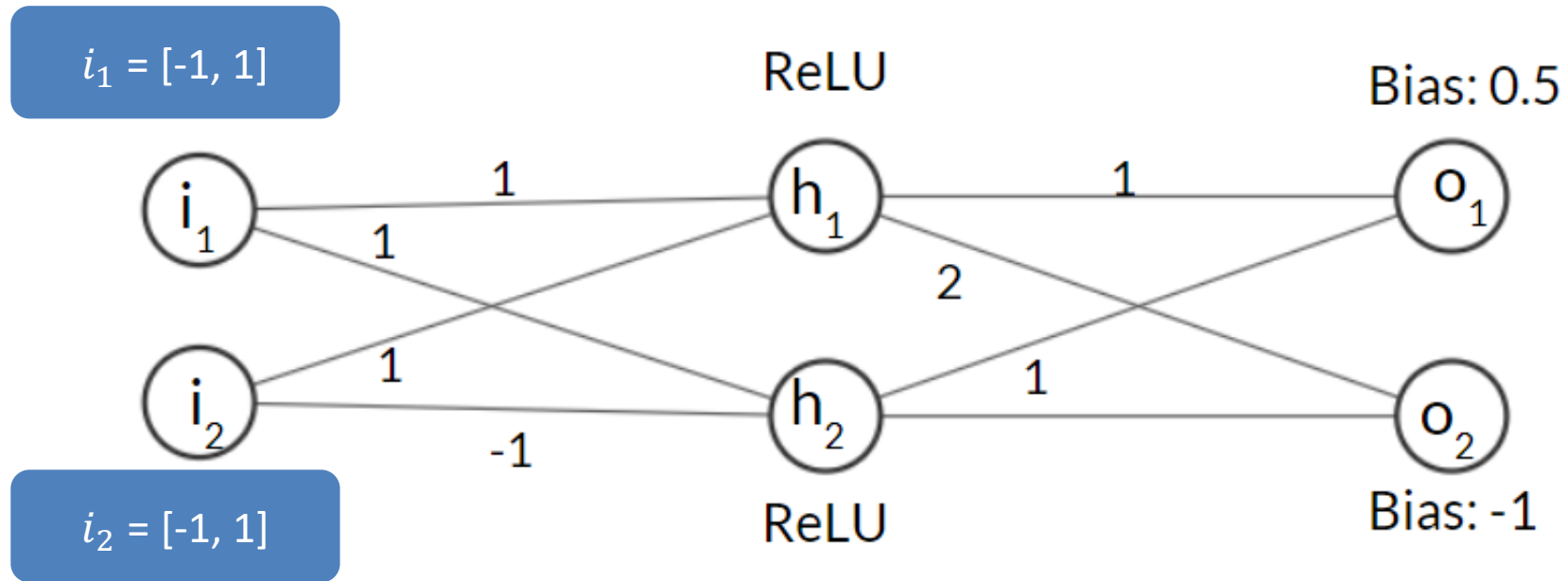# Problem Statement

Problem Statement

Collect Data → Train PINNs → Evaluate Model → Testing

No guarantees ☹

Problem Statement

Collect Data → Train PINNs → Evaluate Model

Evaluate Model → Testing
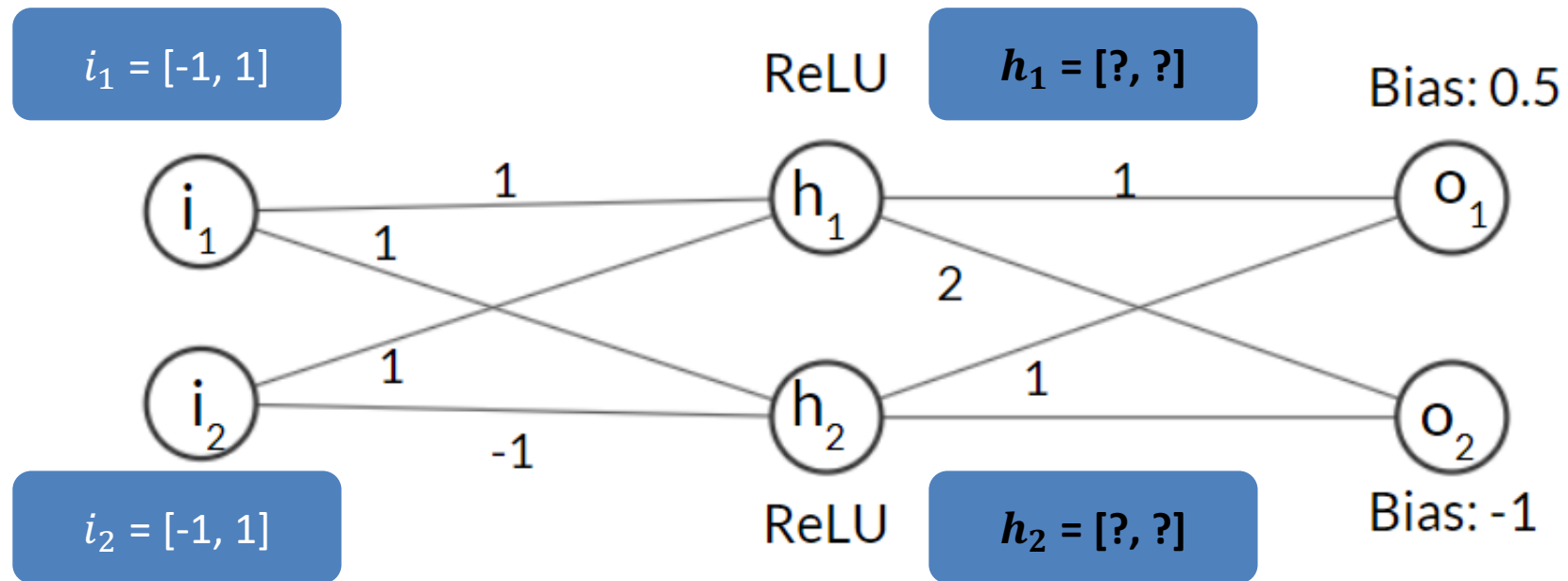No guarantees ☹

Evaluate Model → Verification
Guaranteed ☺

Testing
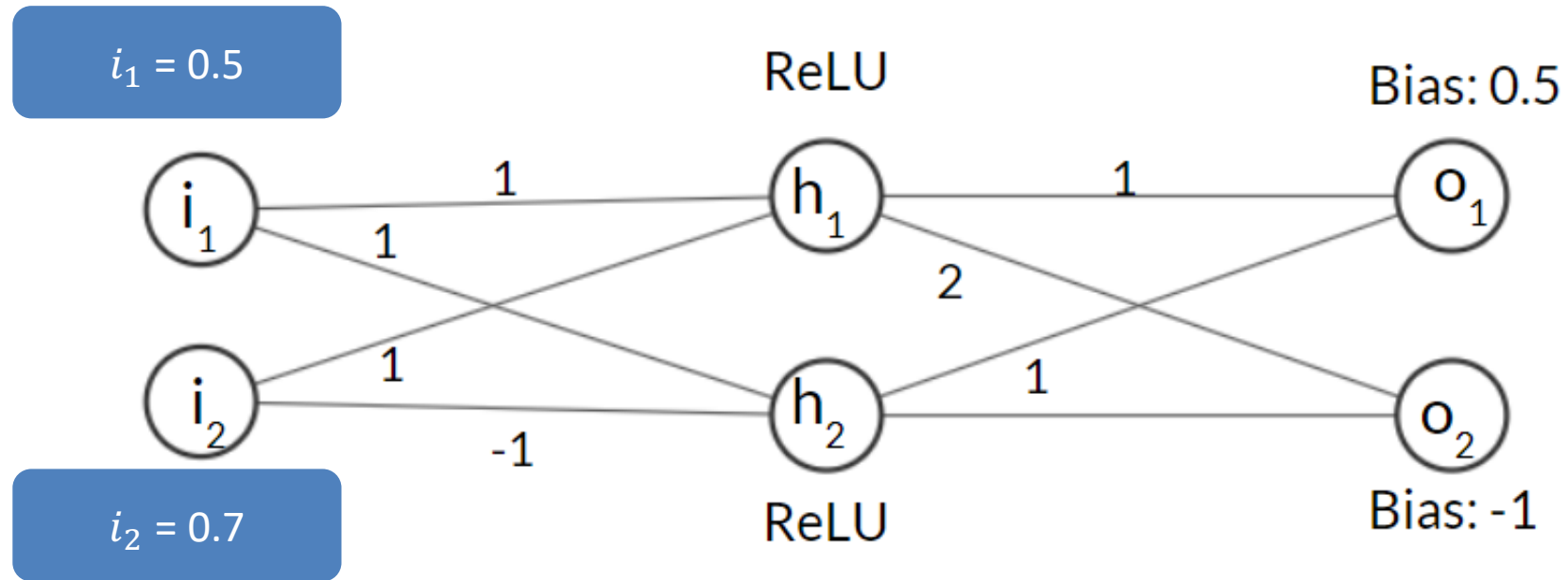
Verification

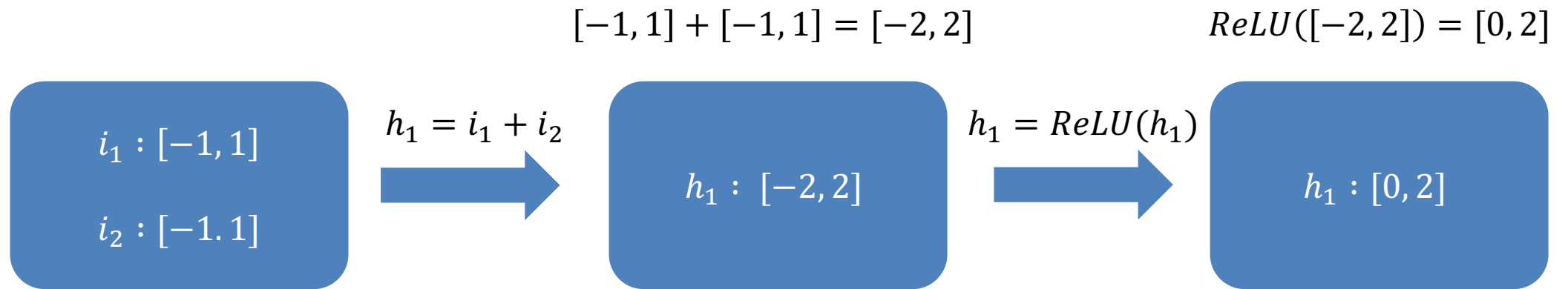# Verification

Verification

Interval Arithmetic

$$i_1 : [-1, 1]$$

$$i_2 : [-1.1]$$

$$h_1 = i_1 + i_2$$

$$h_1 : [?, ?]$$

$$h_1 = ReLU(h_1)$$

$$h_1 : [?, ?]$$

Interval Arithmetic

$$[-1, 1] + [-1, 1] = [-2, 2]$$

$i_1 : [-1, 1]$

$i_2 : [-1. 1]$

$h_1 = i_1 + i_2$

$h_1 : [-2, 2]$

$h_1 = ReLU(h_1)$

$h_1 : [?, ?]$

Interval Arithmetic

$$[-1,1] + [-1,1] = [-2,2]$$

$$ReLU([-2,2]) = [0,2]$$

$i_1 : [-1,1]$

$i_2 : [-1.1]$

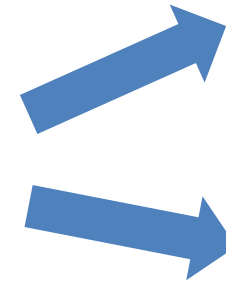$h_1 = i_1 + i_2$

$h_1 : [-2,2]$

$h_1 = ReLU(h_1)$

$h_1 : [0,2]$

Residual Error

Finding upper bound of $u_t + u \cdot u_x - \mu \cdot u_{xx}$
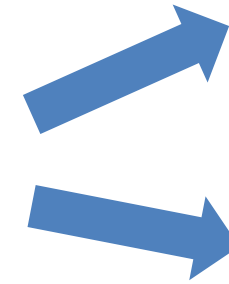
Interval
Arithmetic

+

Chain
Rule

Residual Error

Finding upper bound of $u_t + u \cdot u_x - \mu \cdot u_{xx}$
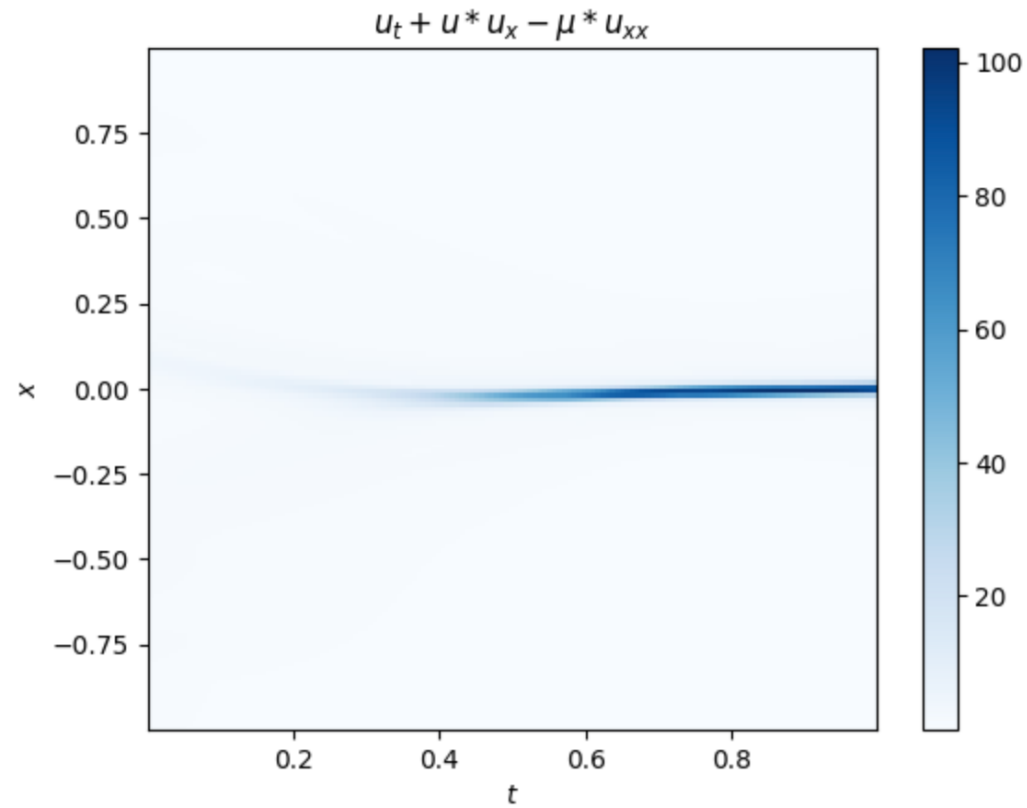

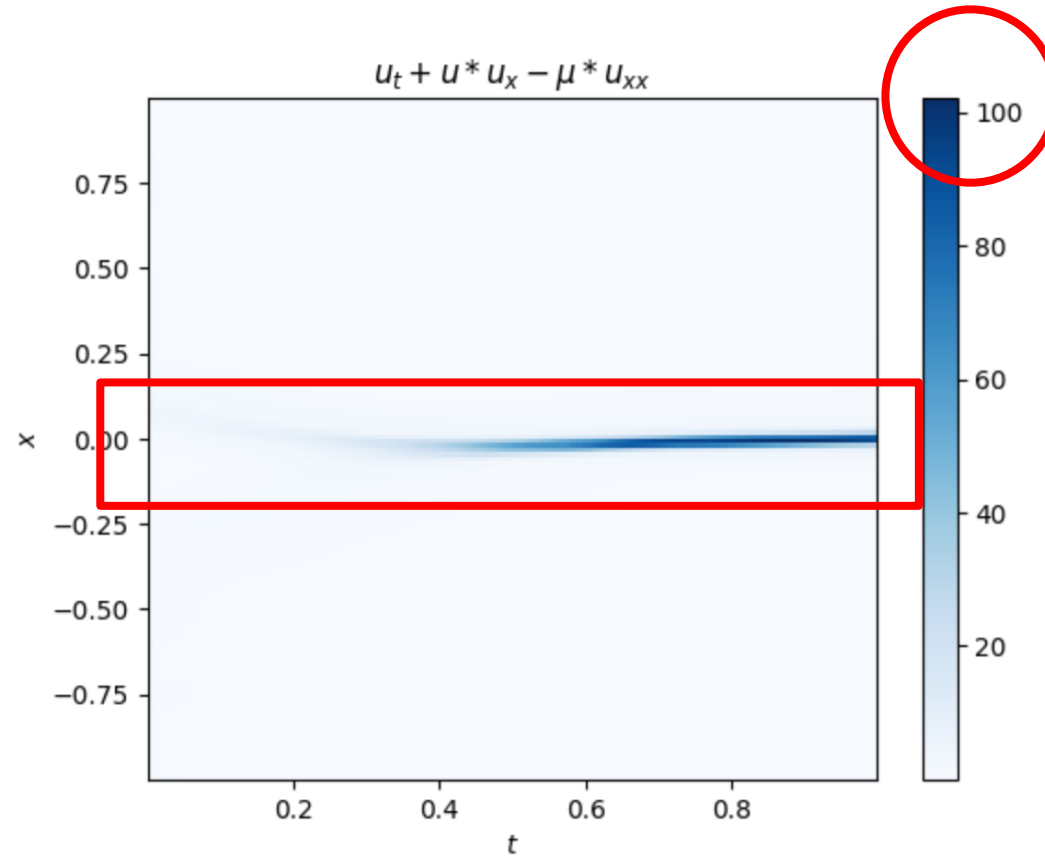Interval
Arithmetic

+

Chain
Rule

Fully connected, ReLU, tanh

- Intervals,
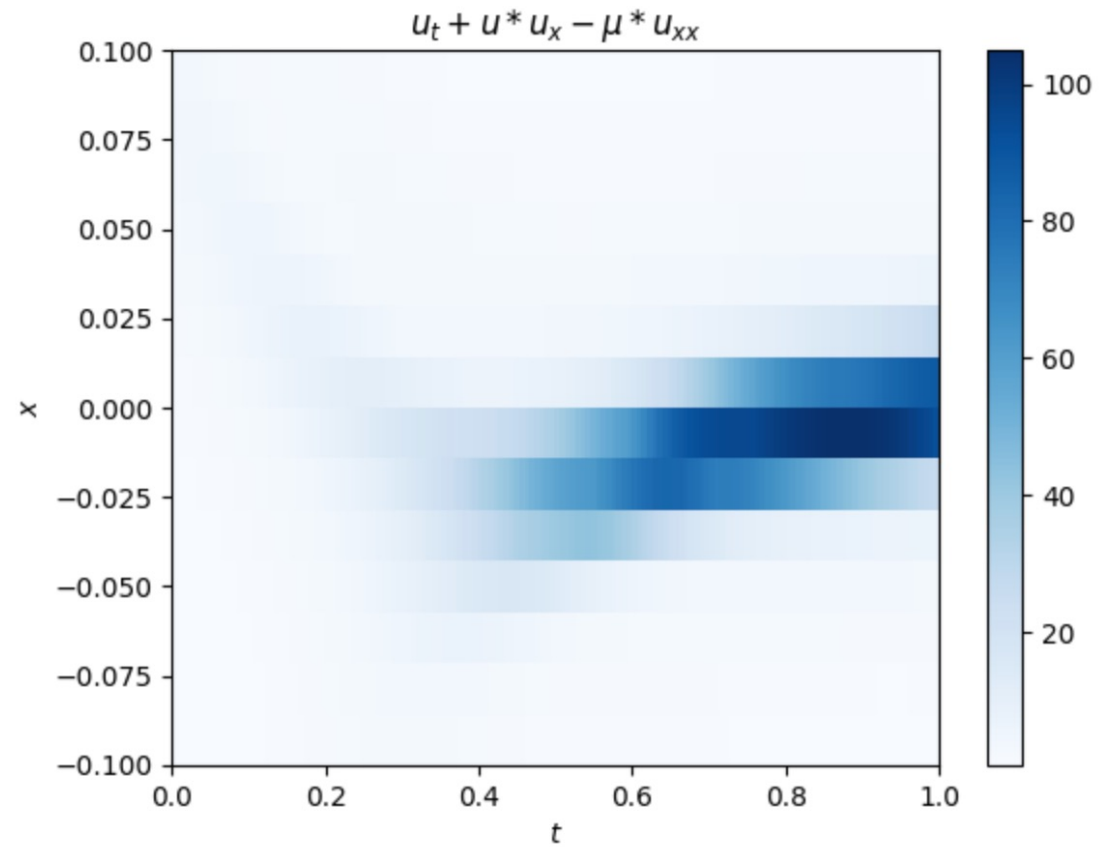- Derivative Intervals
- Double Derivative Intervals

Results – Residual Error

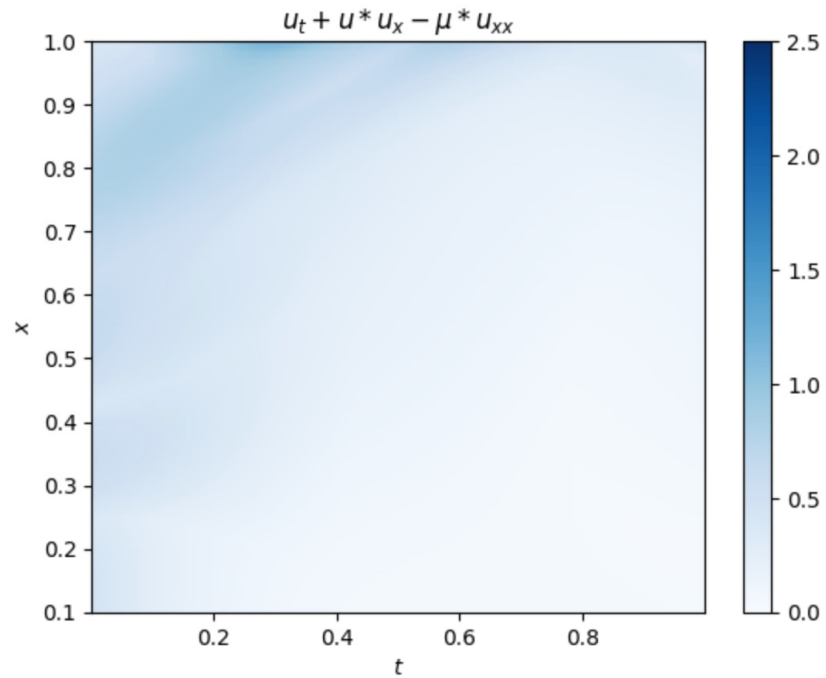$$u_t + u * u_x - \mu * u_{xx}$$

Results – Residual Error

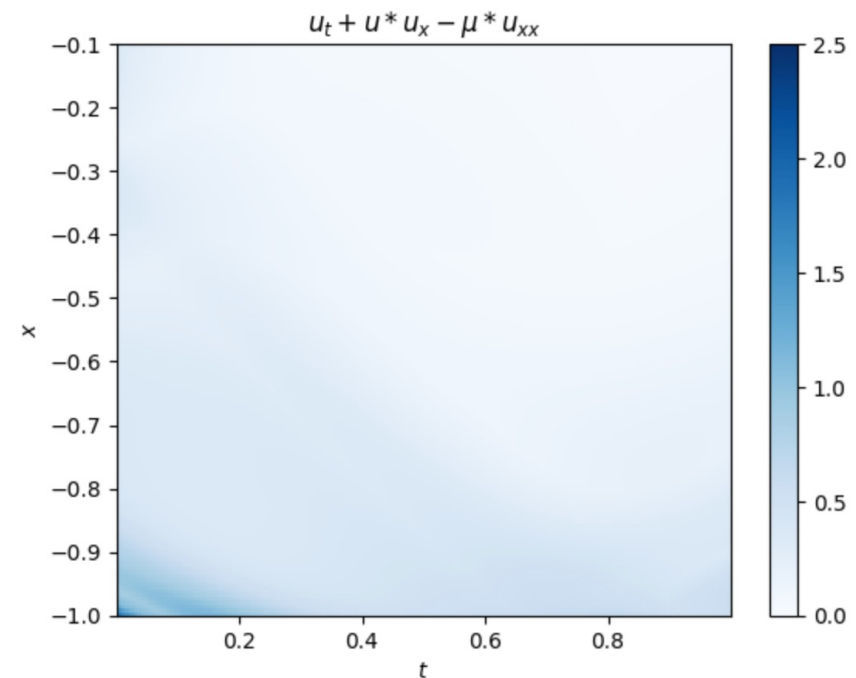$$u_t + u * u_x - \mu * u_{xx}$$

Results – Residual Error
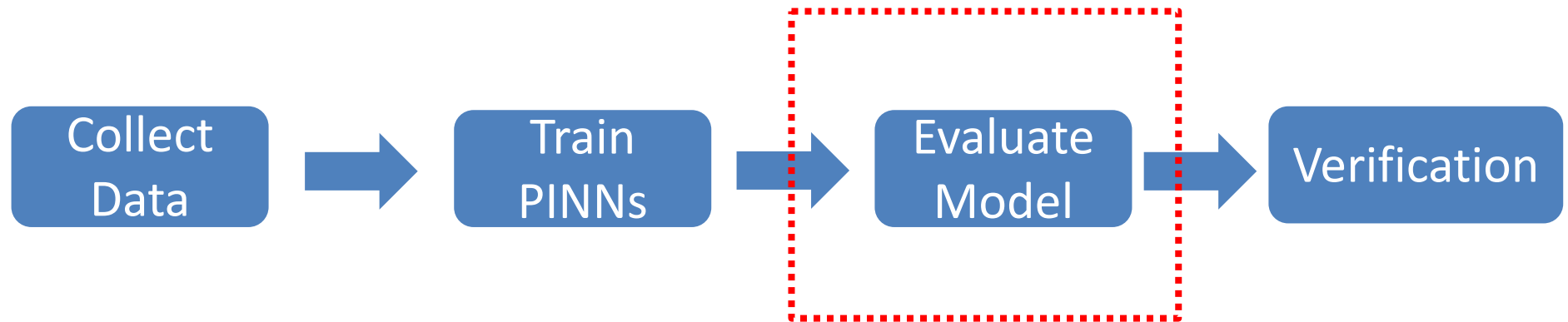
Results – Residual Error
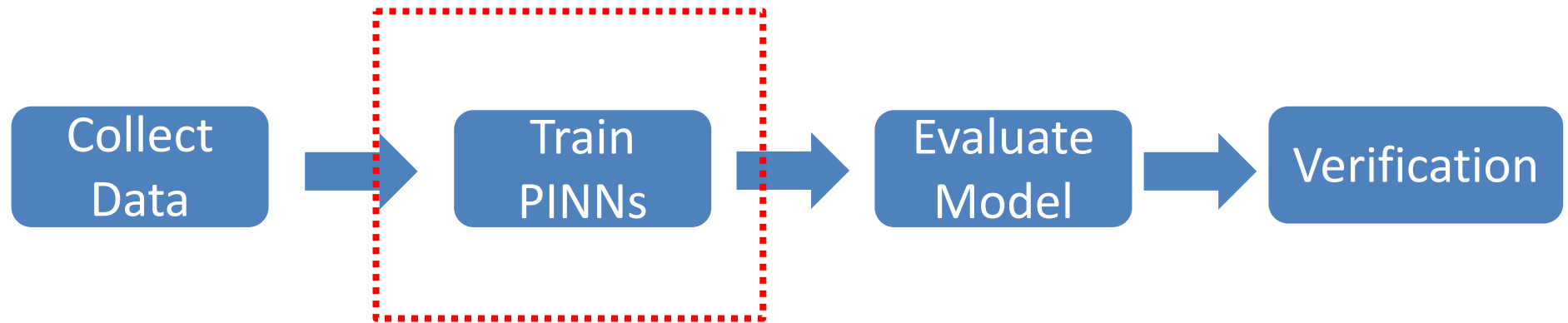

$$x \in [0.1, 1]$$

$$x \in [-1, -0.1]$$

What Now?

Improved Training

Standard Training

Random points sampled from input space

Adversarial Training
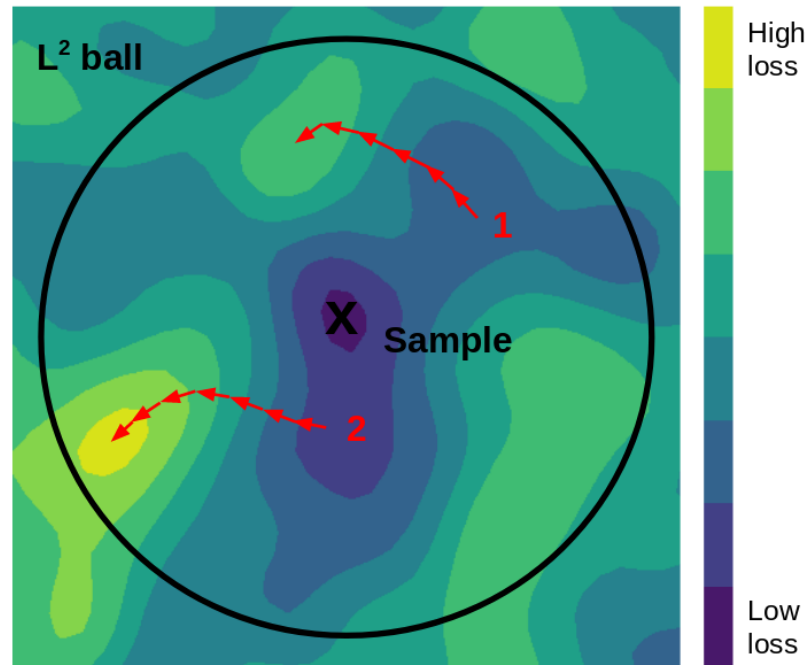
Pick worst performing point in a local region

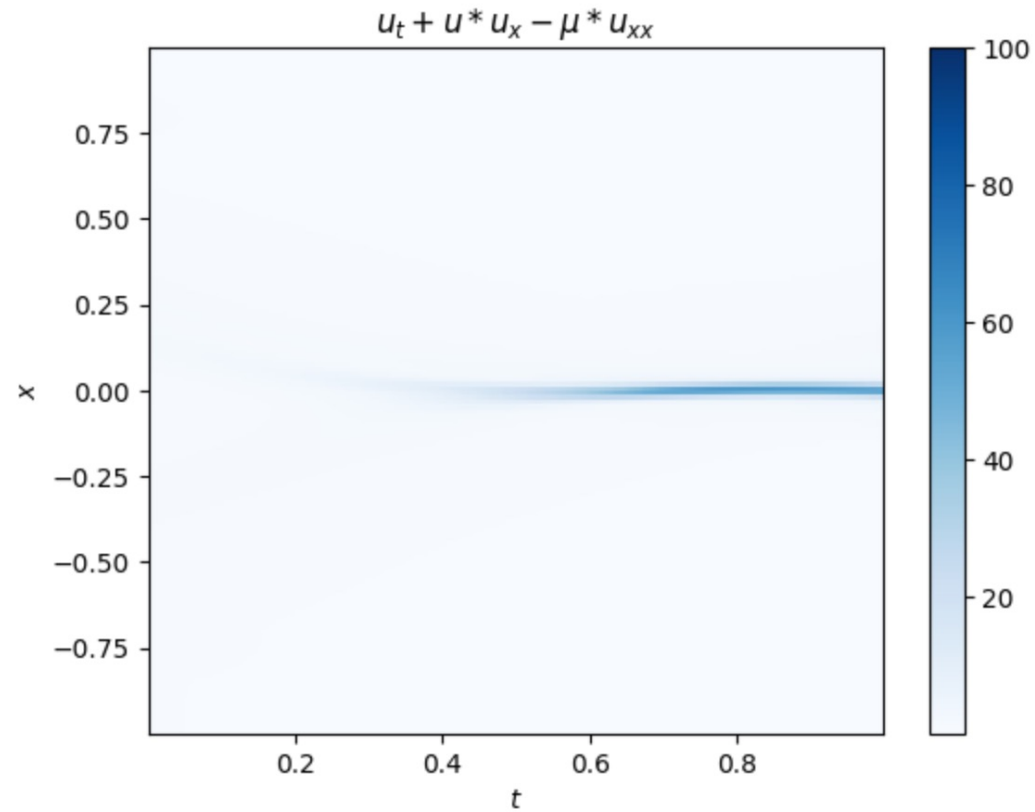# Adversarial Training

Pick worst performing point in a local region

Adversarial !!!
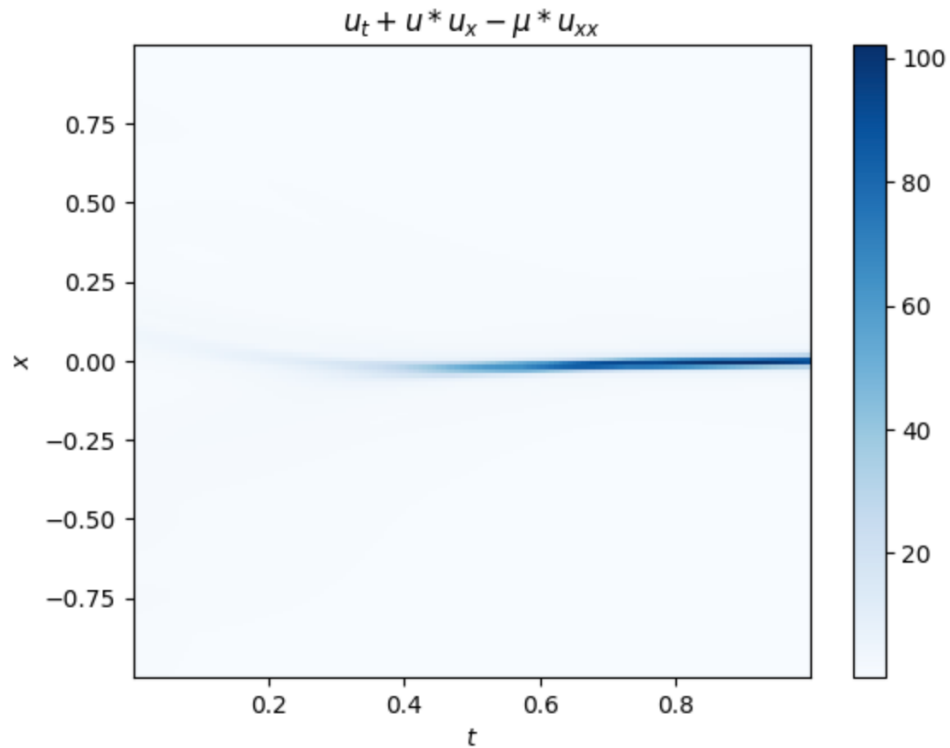
Finding Adversarial Points - PGD
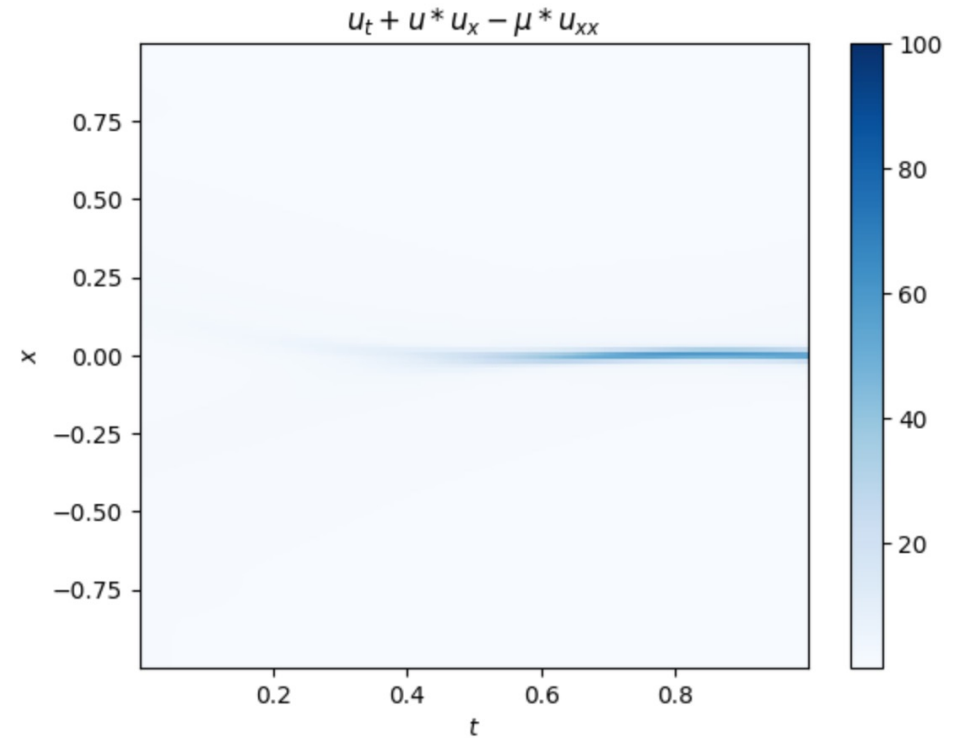
Projected Gradient Descent

Residual Error – Adversarial Training
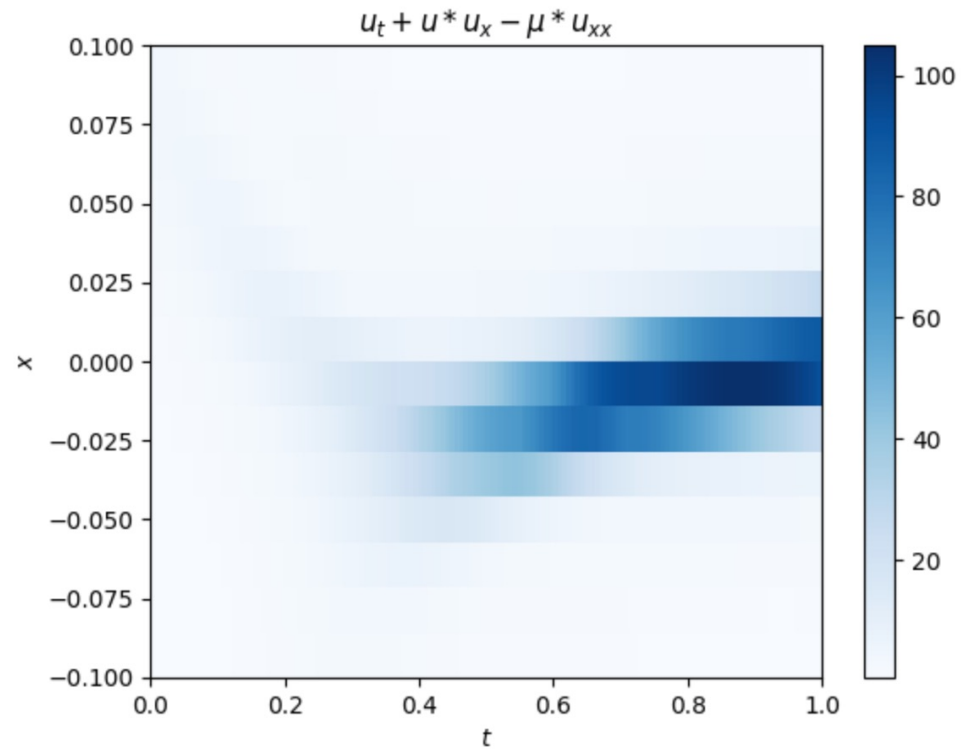
Residual Error - Comparison
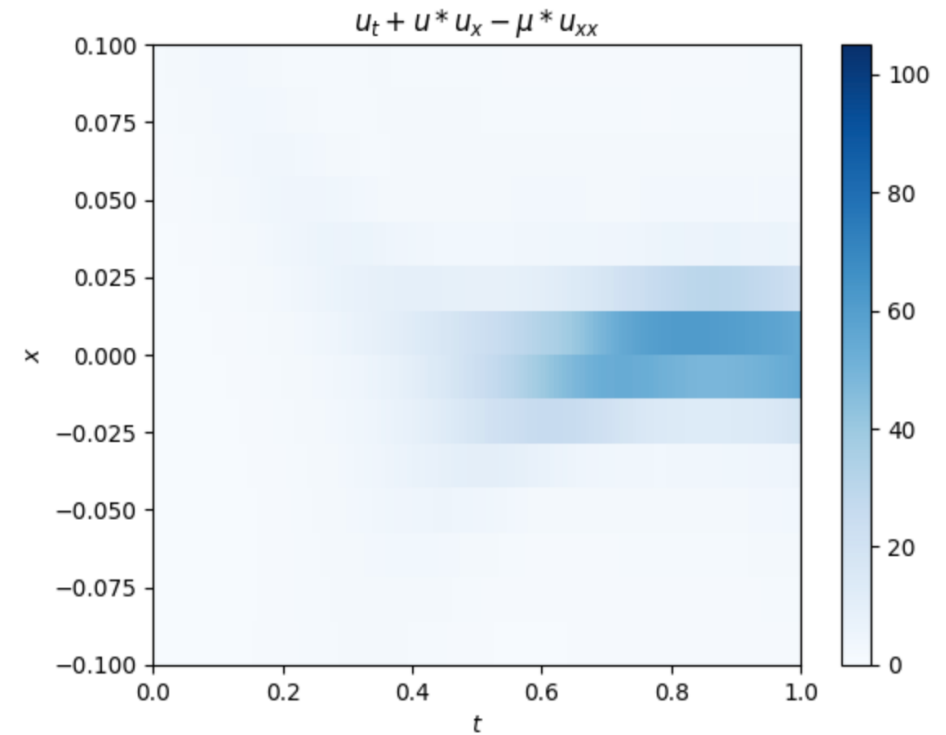

Standard Training

Adversarial Training

Residual Error - Comparison

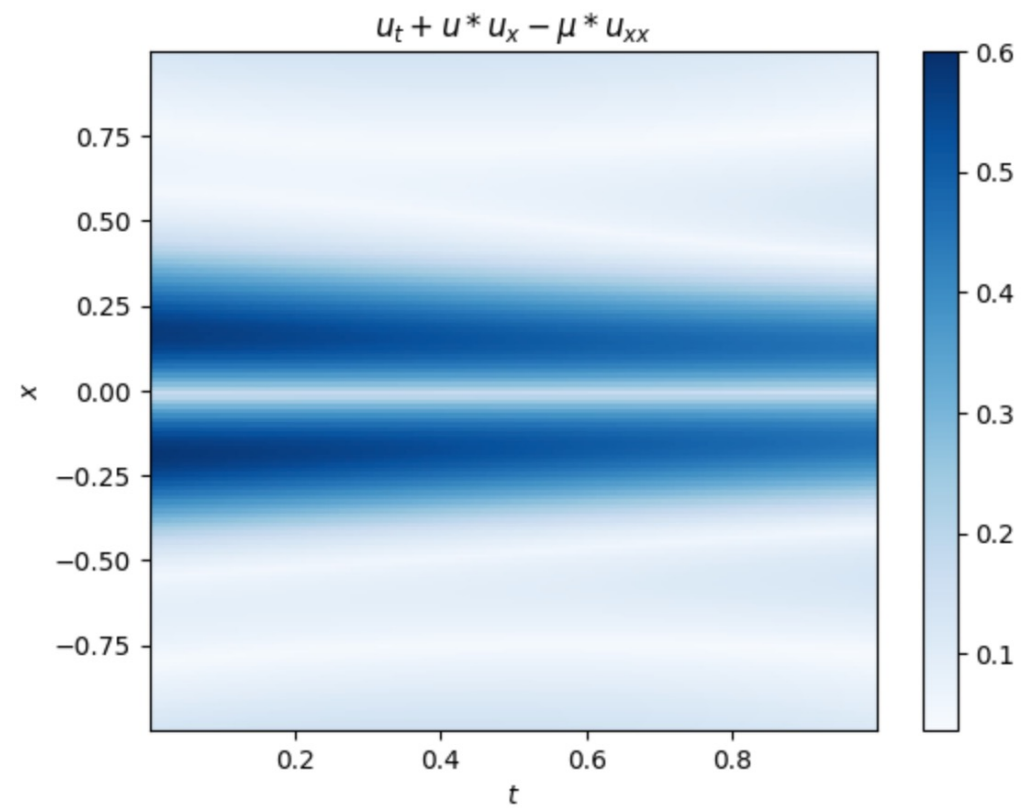Standard Training

Adversarial Training
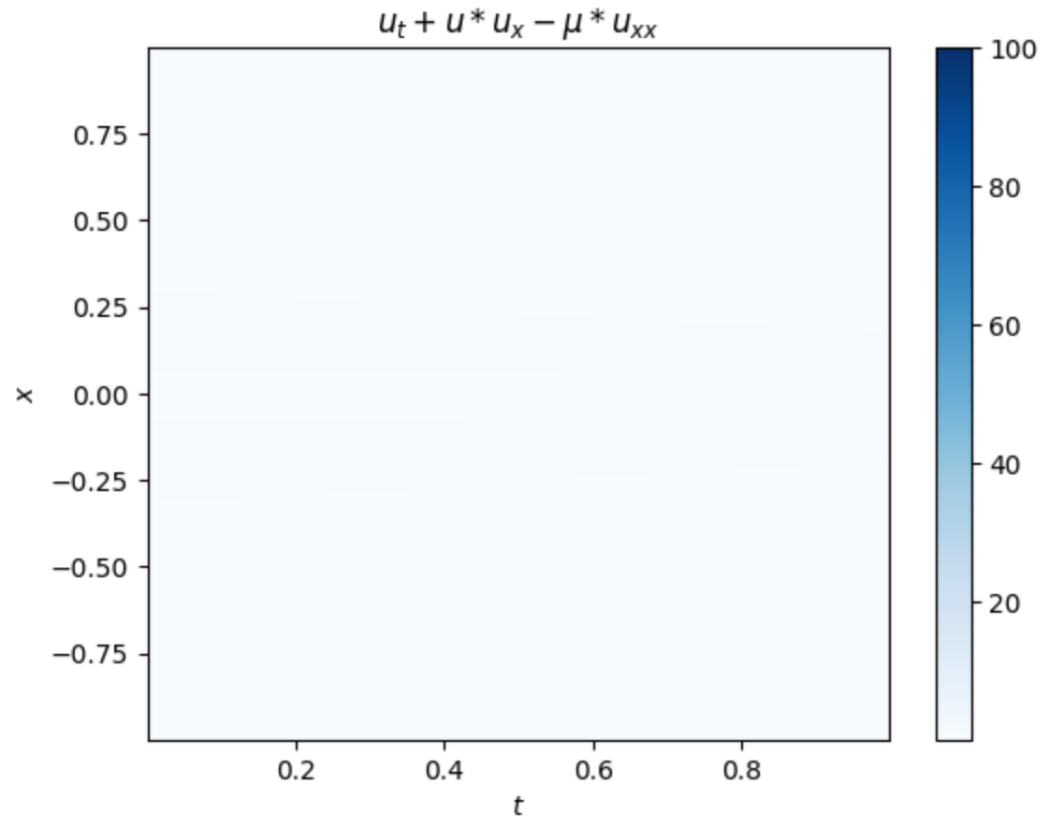
Certified Training

Over-approximated loss of a local region

Certified Training

Over-approximated loss of a local region

Interval arithmetic

Residual error

Certified Training

$u(x, t)$